



## Use of Unknown Weight to Enhance Ratio-Type Estimator in Simple Random Sampling

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### ABSTRACT

In this paper, a ratio-type estimator of finite population mean has been proposed. The proposed estimator was obtained by transforming Gupta and Yadav (2018) estimator. The main purpose of this study is to develop a new and efficient estimator to improve the precision of estimation of population mean in sample random sampling without replacement. The properties of the proposed estimator namely Bias and Mean Square Error were obtained up to first order of approximation using Taylor's Series expansion and the condition for its efficiency over some existing estimators was also established. The efficiency of proposed estimator exhibits significant improvement over the estimators considered in the study. The results of the empirical study show that the proposed estimator is more efficient than existing estimators based on the comparison criteria measures.

**Keywords:** Unknown- weight, Efficiency, Mean Square Error

### INTRODUCTION

The use of auxiliary information in probability sampling has been a commonly used device for improving the precision of the estimator of finite population mean or total under study. If use intelligibly, this information provides us with the sampling strategies better than those in which no auxiliary information is used. Cochran (1940) initiated the use of auxiliary information and proposed a ratio estimator for population mean. It is so far established fact that the ratio type estimator provides better efficiency in comparison to simple mean estimator provided the study and auxiliary variables are positively correlated. If the correlation between the study and auxiliary variables is negative, product estimator is more efficient than the sample mean estimator. Modified ratio estimators came into existence and were constructed by using one or more unknown constants. In a class of estimators, the estimator with minimum variance or mean square error is regarded as the most efficient estimator (Muillet *al.* 2018).

One of the techniques of improving and modifying estimator in survey sample is the use of unknown weight ( $\alpha$ ). Unknown weight is a

suitable constant which minimizes the mean square error of an estimator of population mean by the process called partially differentiation in order to be known. Its ranges from 0 – 1. Unknown weight is a nonnegative value which determinethe mean square error.

The purpose of this study is to develop new and efficient estimator to improve the precision of estimation of population mean in sample random sampling without replacement.

Let  $U = \{U_1, U_2, U_3, \dots, U_N\}$  be a finite population having  $N$  units and each  $U_i = (X_i, Y_i)$ ,  $i = 1, 2, 3, \dots, N$  has a pair of values.  $Y$  is the study variable and  $X$  is the auxiliary variable which is correlated with  $Y$ . Let  $y = \{y_1, y_2, \dots, y_n\}$  and  $x = \{x_1, x_2, \dots, x_n\}$  be  $n$  sample values.  $\bar{y}$  and  $\bar{x}$  are the sample means of

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the study and auxiliary variables respectively. Let  $S_y^2$  and  $S_x^2$  be the finite population variance of  $Y$  and  $X$  respectively and  $s_y^2$  and  $s_x^2$  be respective sample variances based on the random sample of size  $n$  drawn without replacement. The following are the notations used in the study.

- $N$ : Population size
  - $n$ : Sample size
  - $Y$ : Study variable
  - $X$ : Auxiliary variable
  - $\bar{y}, \bar{x}$ : Sample means of study and auxiliary variables
  - $\bar{Y}, \bar{X}$ : Population means of study and auxiliary variables
  - $\rho$ : Coefficient of correlation
  - $C_y, C_x$ : Coefficient of variations of study and auxiliary variables
  - $\beta_1$ : Coefficient of skewness of auxiliary variable
  - $\beta_2$ : Coefficient of kurtosis of auxiliary variable
  - $M_d$ : Median of the auxiliary variable
  - $e_0$ : Error term associated with the study variable
  - $e_1$ : Error term associated with the auxiliary variable
- $$s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2, \quad S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2,$$
- $$S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2$$

The ways of developing efficient estimators has been discussed by many researchers to contribute their quota to the estimation of finite population mean or variance. Murthy (1967) proposed a product type estimator to estimate the population of study variable by the used of auxiliary information when the coefficient of correlation is negative. Singh and Tailor (2003) developed a family of estimators using known values of some parameters to estimate the population mean of the study variable. Abid *et al.* (2016) proposed a class of ratio estimators incorporated non-conventional location parameters for the estimation of population mean. Other researchers are Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Yan and Tian (2010), Subramani and

Kumarapandiyan (2012, 2013), Yadav *et al.* (2016), Subzar *et al.* (2017), Muili *et al.* (2018), Muili *et al.* (2019a, 2019b, 2019c, 2019d, 2019e), Muili and Audu (2019),. Audu *et al.* (2020a, 2020b), Muili *et al.* (2020), etc.

The usual estimator for population mean ( $\bar{Y}$ ), its the corresponding sample mean  $\bar{y}$  given as:

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \tag{1}$$

It is unbiased for population mean and its variance is given as:

$$V(\bar{y}) = \gamma \bar{Y}^2 C_y^2 \tag{2}$$

Cochran (1940) proposed the usual ratio estimator for estimating population mean ( $\bar{Y}$ ) of the study variable ( $Y$ ) as:

$$\hat{Y}_r = \frac{\bar{y}}{\bar{x}} \bar{X} \tag{3}$$

$$Bias(\hat{Y}_r) = \gamma \bar{Y} (C_x^2 - \rho C_x C_y) \tag{4}$$

$$MSE(\hat{Y}_r) = \gamma \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_y C_x) \tag{5}$$

Sisodia and Dwivesdi (1981) proposed ratio type estimator of population mean based on the value of coefficient of variation ( $C_x$ ) of auxiliary variable given as:

$$\hat{Y}_1 = \bar{y} \left( \frac{\bar{X} + C_x}{\bar{x} + C_x} \right) \tag{6}$$

Upadhyaya and Singh (1999) proposed ratio type estimator of population mean by imposing coefficient of variation ( $C_x$ ) and coefficient of kurtosis ( $\beta_2$ ) of auxiliary variable on the work of Sisodia and Dwivesdi (1981) as:

$$\hat{Y}_2 = \bar{y} \left( \frac{\bar{X} C_x + \beta_2}{\bar{x} C_x + \beta_2} \right) \tag{7}$$

Singh and Tailor (2003) incorporated coefficient of correlation ( $\rho$ ) of auxiliary variable in the work of Cochran (1940) and defined a new estimator of population mean as:

$$\hat{Y}_3 = \bar{y} \left( \frac{\bar{X} + \rho}{\bar{x} + \rho} \right) \tag{8}$$

Jerajuddin and Kishun (2016) was the first that used sample size ( $n$ ) in order to increase precision of Cochran (1940) estimator of population mean as:

$$\hat{Y}_7 = \bar{y} \left( \frac{\bar{X} + n}{\bar{x} + n} \right) \tag{9}$$

Gupta and Yadav (2018) proposed a ratio estimator by using linear combination on the

work of Jerajuddin and Kishun (2016) for estimating population mean given as:

$$\hat{Y}_8 = \bar{y} \left[ \alpha + (1-\alpha) \left( \frac{\bar{X} + n}{\bar{x} + n} \right) \right] \tag{10}$$

Table 1: Some Existing Estimators with their Properties

Estimator	Constant	Bias	MSE
$\hat{Y}_1 = \bar{y} \left( \frac{\bar{X} + C_x}{\bar{x} + C_x} \right)$ Sisodia and Dwivedi [1981]	$\delta_1 = \left( \frac{\bar{X}}{\bar{X} + C_x} \right)$	$\gamma \bar{Y} (\delta_1^2 C_x^2 - 2\delta_1 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_1^2 C_x^2 - 2\delta_1 \rho C_x C_y)$
$\hat{Y}_2 = \bar{y} \left( \frac{\bar{X} C_x + \beta_2}{\bar{x} C_x + \beta_2} \right)$ Upadhyaya and Singh [1999]	$\delta_2 = \left( \frac{\bar{X} C_x}{\bar{x} C_x + \beta_2} \right)$	$\gamma \bar{Y} (\delta_2^2 C_x^2 - 2\delta_2 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_2^2 C_x^2 - 2\delta_2 \rho C_x C_y)$
$\hat{Y}_3 = \bar{y} \left( \frac{\bar{X} + \rho}{\bar{x} + \rho} \right)$ Singh and Tailor [2003]	$\delta_3 = \left( \frac{\bar{X}}{\bar{x} + \rho} \right)$	$\gamma \bar{Y} (\delta_3^2 C_x^2 - 2\delta_3 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_3^2 C_x^2 - 2\delta_3 \rho C_x C_y)$
$\hat{Y}_4 = \bar{y} \left( \frac{\bar{X} + \beta_2}{\bar{x} + \beta_2} \right)$ Singh et al [2004]	$\delta_4 = \left( \frac{\bar{X}}{\bar{x} + \beta_2} \right)$	$\gamma \bar{Y} (\delta_4^2 C_x^2 - 2\delta_4 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_4^2 C_x^2 - 2\delta_4 \rho C_x C_y)$
$\hat{Y}_5 = \bar{y} \left( \frac{\bar{X} + \beta_1}{\bar{x} + \beta_1} \right)$ Yan and Tian [2010]	$\delta_5 = \left( \frac{\bar{X}}{\bar{x} + \beta_1} \right)$	$\gamma \bar{Y} (\delta_5^2 C_x^2 - 2\delta_5 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_5^2 C_x^2 - 2\delta_5 \rho C_x C_y)$
$\hat{Y}_6 = \bar{y} \left( \frac{\bar{X} + M_d}{\bar{x} + M_d} \right)$ Subramani and Kumarpandiyan [2013]	$\delta_6 = \left( \frac{\bar{X}}{\bar{x} + M_d} \right)$	$\gamma \bar{Y} (\delta_6^2 C_x^2 - 2\delta_6 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_6^2 C_x^2 - 2\delta_6 \rho C_x C_y)$
$\hat{Y}_7 = \bar{y} \left( \frac{\bar{X} + n}{\bar{x} + n} \right)$ Jerajuddin and Kishun [2016]	$\delta_7 = \left( \frac{\bar{X}}{\bar{x} + n} \right)$	$\gamma \bar{Y} (\delta_7^2 C_x^2 - 2\delta_7 \rho C_x C_y)$	$\gamma \bar{Y}^2 (C_y^2 + \delta_7^2 C_x^2 - 2\delta_7 \rho C_x C_y)$
$\hat{Y}_8 = \bar{y} \left[ \alpha + (1-\alpha) \left( \frac{\bar{X} + n}{\bar{x} + n} \right) \right]$ Gupta and Yadav [2018]	$\delta_8 = \left( \frac{\bar{X}}{\bar{x} + n} \right)$	$\gamma \bar{Y} \left( \delta_8^2 C_x^2 - \delta_8 \rho C_x C_y + \alpha \delta_8 \rho C_x C_y - \alpha \delta_8^2 C_x^2 \right)$	$\gamma \bar{Y}^2 \left[ C_y^2 + \delta_8^2 C_x^2 - 2\delta_8 \rho C_x C_y - \frac{(\delta_8^2 C_x^2 - \delta_8 \rho C_x C_y)^2}{\delta_8^2 C_x^2} \right]$

The existing estimators with their properties are shown in Table 1

**METHODOLOGY**

**The Proposed Estimator**

The proposed ratio type estimator of finite population mean is given as:

$$\hat{Y}_p = \bar{y} \alpha \left( \frac{\bar{X} + n}{\bar{x} + n} \right) \tag{11}$$

where  $\alpha$  is unknown weight to be determined such that the MSE of the proposed estimator ( $\hat{Y}_p$ ) is minimized.

The subject of validity (agreement to acceptability) and reliability (dependable) are complex and controversial but of great

importance in research methodology and any research depend strongly on the methods used at various stages of the design of the survey, collection of data, analysis and interpretation of the results. Taylor’s Expansion was used for the derivation of properties (Bias, Mean Square Error and Minimum Mean Square Error) of the proposed estimator.

This study focused on measure of precision, which is a function of the variance for the estimator under the simple random sampling without replacement (SRSWOR). It also, centered on checking precision on estimate and percentage gain in relative efficiency.

**Assumptions of the Proposed Estimator**

- i. Population of study is finite.
- ii. Sample size is known.
- iii. Information about the study variable  $(y)$  and auxiliary variable  $(x)$  in sample are completely available.
- iv. The precision of the estimation is determined by evaluating the efficiency of the relative gain in information from the sample.

In order to obtain the bias and MSE, we define

$$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \text{ and } e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}} \text{ such that}$$

$$\bar{y} = \bar{Y}(1 + e_0) \text{ and } \bar{x} = \bar{X}(1 + e_1)$$

$$\left. \begin{aligned} E(e_0) = E(e_1) = 0, E(e_0^2) = \gamma C_y^2, \\ E(e_1^2) = \gamma C_x^2, E(e_0 e_1) = \gamma \rho C_y C_x \end{aligned} \right\} \quad (12)$$

Now expressing (11) in error terms as:

$$\hat{Y}_p = \bar{Y} \alpha (1 + e_0) \left( \frac{\bar{X} + n}{\bar{X}(1 + e_1) + n} \right) \quad (13)$$

Simplifying (13) up to first order approximation, it reduces to (14) as:

$$\hat{Y}_p = \bar{Y} \alpha (1 + e_0) \left( \frac{\bar{X} + n}{(\bar{X} + n) \left( 1 + \frac{\bar{X} e_1}{\bar{X} + n} \right)} \right) \quad (14)$$

$$\hat{Y}_p = \bar{Y} \alpha (1 + e_0) \left( \frac{1}{\left( 1 + \frac{\bar{X} e_1}{\bar{X} + n} \right)} \right) \quad (15)$$

$$\hat{Y}_p = \bar{Y} \alpha (1 + e_0) \left( \frac{1}{(1 + \delta e_1)} \right) \quad (16)$$

Where  $\delta = \frac{\bar{X}}{\bar{X} + n}$

$$\hat{Y}_p = \bar{Y} \alpha (1 + e_0) (1 + \delta e_1)^{-1} \quad (17)$$

Neglecting the term of order more than two because we restricted to first order approximation

$$\hat{Y}_p = \bar{Y} (\alpha + \alpha e_0) (1 - \delta e_1 + \delta^2 e_1^2) \quad (18)$$

$$\hat{Y}_p = \bar{Y} (\alpha + \alpha e_0 - \alpha \delta e_1 - \alpha \delta e_0 e_1 + \alpha \delta^2 e_1^2 + \alpha \delta^2 e_0 e_1^2) \quad (19)$$

Neglecting the term of order more than two, (19) gives

$$\hat{Y}_p = \bar{Y} (\alpha + \alpha e_0 - \alpha \delta e_1 - \alpha \delta e_0 e_1 + \alpha \delta^2 e_1^2) \quad (20)$$

Subtracting  $\bar{Y}$  from both sides of (18)

$$\hat{Y}_p - \bar{Y} = \bar{Y} (\alpha + \alpha e_0 - \alpha \delta e_1 - \alpha \delta e_0 e_1 + \alpha \delta^2 e_1^2) - \bar{Y} \quad (21)$$

$$\hat{Y}_p - \bar{Y} = \bar{Y} ((\alpha - 1) + \alpha e_0 - \alpha \delta e_1 - \alpha \delta e_0 e_1 + \alpha \delta^2 e_1^2) \quad (22)$$

Taking Expectation of both sides of (22)

$$E(\hat{Y}_p - \bar{Y}) = \bar{Y} E((\alpha - 1) + \alpha e_0 - \alpha \delta e_1 - \alpha \delta e_0 e_1 + \alpha \delta^2 e_1^2) \quad (23)$$

Applying the results of (12) obtaining the Bias  $(\hat{Y}_p)$  as:

$$Bias(\hat{Y}_p) = \bar{Y} ((\alpha - 1) + \gamma \alpha \delta^2 C_x^2 - \alpha \gamma \delta C_y C_x) \quad (24)$$

Squaring both sides of (20), Expanding and taking expectation gives

$$MSE(\hat{Y}_p) = \bar{Y}^2 E \left[ \begin{aligned} &(\alpha - 1)^2 + \alpha^2 e_0^2 + \alpha^2 \delta^2 e_1^2 + 2\alpha(\alpha - 1)e_0 - 2\alpha\delta(\alpha - 1)e_1 \\ &- 2\alpha\delta(\alpha - 1)e_0 e_1 + 2\alpha\delta^2(\alpha - 1)e_1^2 - 2\alpha^2\delta e_0 e_1 \end{aligned} \right] \quad (25)$$

Applying the results of (12), obtaining  $MSE(\hat{Y}_p)$  as:

$$MSE(\hat{Y}_p) = \bar{Y}^2 \left[ \begin{aligned} &(\alpha - 1)^2 + \gamma \alpha^2 C_y^2 + (3\gamma \alpha^2 \delta^2 - 2\gamma \alpha \delta^2) C_x^2 \\ &+ (2\gamma \alpha \delta - 4\gamma \alpha^2 \delta) \rho C_x C_y \end{aligned} \right] \quad (26)$$

Taking the partial derivative of (24) with respect to  $\alpha$  and equate to zero gives

$$\frac{\partial (MSE(\hat{Y}_p))}{\partial \alpha} = \bar{Y}^2 \left[ \begin{aligned} &2(\alpha - 1) + 2\gamma \alpha C_y^2 + (6\gamma \alpha \delta^2 - 2\gamma \delta^2) C_x^2 \\ &+ (2\gamma \delta - 8\gamma \alpha \delta) \rho C_x C_y \end{aligned} \right] = 0 \quad (27)$$

Simplifying (27) for  $\alpha$ , obtaining optimum value of  $\alpha$  as:

$$\alpha^{opt} = \frac{1 + \gamma \delta^2 C_x^2 - \gamma \delta \rho C_x C_y}{1 + \gamma C_y^2 + 3\gamma \delta^2 C_x^2 - 4\gamma \delta \rho C_x C_y} \quad (28)$$

Substitute (28) in (26), gives

$$MSE(\hat{Y}_p)_{min} = \bar{Y}^2 \left[ 1 - \frac{(1 + \gamma \delta^2 C_x^2 - \gamma \delta \rho C_x C_y)^2}{1 + \gamma C_y^2 + 3\gamma \delta^2 C_x^2 - 4\gamma \delta \rho C_x C_y} \right] \quad (29)$$

Table 2: Populations I and II: Murthy (1967), Population III: Mukhopadhyay (2009) Population IV: Singh and Chaudhary (1986)

Parameter	Population I	Population II	Population III	Population IV
$N$	80	40	40	34
$n$	20	8	8	20
$\bar{Y}$	51.8264	50.7858	50.7858	856.4117
$\bar{X}$	11.2646	2.3033	9.4543	208.8823
$\rho$	0.9413	0.8006	0.8349	0.4491
$C_y$	0.3542	0.3295	0.3295	0.8561
$C_x$	0.7507	0.8406	0.6756	0.7205
$\beta_1$	1.0500	0.9740	0.8799	0.9782
$\beta_2$	-0.0634	-0.5344	-0.4622	0.0978
$M_d$	7.5750	1.2500	7.0700	150

Table 3: Bias of the Estimators

Estimator	Population I	Population II	Population III	Population IV
$\hat{Y}_r$	0.6088	2.4624	1.3742	4.268817
$\hat{Y}_1$	0.0506	0.2760	0.2573	-0.6447285
$\hat{Y}_2$	0.1134	3.7353	0.6588	-0.6210296
$\hat{Y}_3$	0.0350	0.3047	0.2225	-0.6337635
$\hat{Y}_4$	0.1156	3.1516	0.5777	-0.6194826
$\hat{Y}_5$	0.1170	0.1896	0.2131	-0.6550856
$\hat{Y}_6$	-0.1901	0.0479	-0.3213	-2.584918
$\hat{Y}_7$	-0.2083	-0.3242	-0.3424	-1.291627
$\hat{Y}_8$	-0.0408	-0.1017	0.1269	1.851139
$\hat{Y}_p$ Proposed	-0.00267	-0.0390	0.0943	4.78611

Table 4: Mean Square Error (MSE)

Estimator	Population I	Population II	Population III	Population IV
$\hat{Y}_r$	18.9793	95.8641	49.8536	10539.96
$\hat{Y}_1$	15.2581	42.0198	41.0685	10514.92
$\hat{Y}_2$	18.5128	217.7034	61.4577	10535.21
$\hat{Y}_3$	14.4503	43.47728	39.3031	10524.31
$\hat{Y}_4$	18.6279	188.0582	57.3389	10536.54
$\hat{Y}_5$	18.7015	37.6296	38.8230	10506.05
$\hat{Y}_6$	2.7825	30.4328	11.6863	8853.316
$\hat{Y}_7$	1.8289	11.5391	10.6117	9960.905
$\hat{Y}_8$	1.4400	10.0540	8.4831	8834.942
$\hat{Y}_p$ Proposed	0.70512	5.1692	0.3371	592.7365

Table 2 shows the numerical values of the parameters used in computing Bias, Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) of the proposed and other existing estimators considered in the study and the results are presented in tables 3 and 4. Table 3 shows the bias of the proposed and other existing estimators. Table 4 shows the results of Mean Square Errors (MSEs) of the proposed and some existing estimators considered in the study for all the data sets I, II, III and IV. The results showed that the proposed estimator has minimum MSE.

**Percentage Relative Efficiency (PRE)**

Percentage Relative Efficiency (PRE) is the performance measures of estimators efficiency gain. PRE of the estimators were computed see Jerajuddin and Kishun (2016).

here  $\hat{Y}_i$  and  $\hat{Y}_r$  are the existing and proposed estimators in the study

Table 5: Percentage Relative Efficiency (PRE)

Estimator	Population I	Population II	Population III	Population IV
$\hat{Y}_r$	100	100	100	100
$\hat{Y}_1$	66.58149	228.1403	121.3913	100.2381
$\hat{Y}_2$	82.81962	44.03427	81.11856	100.0451
$\hat{Y}_3$	68.25926	220.4924	126.8439	100.1487
$\hat{Y}_4$	87.4494	50.97576	86.94551	100.0325
$\hat{Y}_5$	67.83749	254.7572	128.4125	100.3228
$\hat{Y}_6$	67.57052	315.0026	426.5987	119.051
$\hat{Y}_7$	454.1491	830.7762	469.7984	105.8133
$\hat{Y}_8$	690.9454	953.4921	587.6814	119.2986
$\hat{Y}_p$ Proposed	877.5486	1854.525	14788.96	1778.186

Table 5 shows the Percentage Relative Efficiency (PRE) of the proposed and some existing estimators considered in the study for all the data sets I, II, III and IV. The results revealed that the proposed estimator has highest PRE than other existing estimators.

**Efficiency Comparisons**

In this section efficiencies of the proposed estimator is compared with efficiencies of some existing estimators in the literature.

The  $\hat{Y}_p$  of estimator of population mean is more efficient than  $\hat{Y}_r$  if,

$$MSE(\hat{Y}_p) < MSE(\hat{Y}_r) \tag{30}$$

The proposed estimator ( $\hat{Y}_p$ ) of the population mean is more efficient than  $\hat{Y}_i$  if,

$$MSE(\hat{Y}_p) < MSE(\hat{Y}_i) \quad i = 1, 2, 3, \dots, 8 \tag{31}$$

When conditions (30) and (31) are satisfied, we conclude that the proposed estimator is more efficient than some existing estimators.

**RESULTS AND DISCUSSION**

In this study, we proposed a ratio type population mean estimator and the performance of the proposed estimator over the existing estimators using unknown weight were established. Tables 4 and 5 show the results of Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) of the proposed and some existing estimators considered in the study for all the data sets I, II, III and IV. The results

revealed that the proposed estimator has minimum MSE and highest PRE than other estimators. The results also show that the average dispersion of the proposed estimator gives better estimates on the average than other estimators considered in the study.

### Conclusion

Based on the results in Table 5, the proposed estimator  $(\hat{Y}_p)$  outperformed other existing estimators considered in the study. Therefore, it is clear that the proposed estimator do better than other existing estimators having minimum Mean Square Error (MSE) and the highest Percentage Relative Error (PRE). We therefore recommend the proposed estimator for use in practical application in estimating population mean.

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